Group - A non empty set G, together with a binary composition $*$ is said to form a Group, if satisfies the following conditions

(i) Associativity ⇒ $a * (b * c) = (a * b) * c$ for all $a, b, c \in G$

(ii) Existence of identity ⇒

$\exists$ an element $e \in G$ such that

$a * e = e * a = a$ for all $a \in G$

(where e is called identity)

(iii) Existence of inverse ⇒

For every $a \in G$

$\exists\, a' \in G$ such that

$a * a' = a' * a = e$

(where a' is called inverse of a)

(IV) closure property ⇒

Since $*$ is a binary composition on G, then it is clear that for all $a, b \in G$, $a * b$ is a unique member of G.

ie $a * b \in G \ \forall\ a, b \in G$.

Commutative Group or Abelian Group -

If in addition to the above mentioned conditions G also satisfy the commutative law

ie $a * b = b * a$ for all $a, b \in G$

then G is called an Abelian group or a commutative group.

* Subgroup – A non-empty subset H of a group G is said to be a subgroup of G, if H forms a group under the binary composition of G.

i.e A non empty subset H of a group G is a subgroup of G if

⇔ $a, b \in H \Rightarrow ab \in H$

⇔ $a \in H \Rightarrow a^{-1} \in H$

* Composition Series – A composition series for a group G is a finite sequence of subgroups

$$G = G_0 \triangleright G_1 \triangleright G_2 \triangleright G_3 \cdots\cdots \triangleright G_n = \{e\}$$

⇒ $G_i \triangleright G_{i+1}$ i.e. each subgroup G_{i+1} is a normal subgroup of G_i ($G_{i+1} \triangleleft G_i$)

⇒ The quotient groups G_i / G_{i+1} are simple groups meaning they have no non-trivial normal subgroups. A simple group is one that cannot be broken down further, except for trivial group and itself.